Predicting the entrance opening duration of Intermittently Open/Closed Estuaries (IOCE) in Victoria

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Key Points

- Many estuaries in Victoria close to the ocean and are artificially opened to reduce flood risk
- Their entrances often close again within several days due to high onshore sediment transport
- The geomorphic conditions after lagoon drainage determine the opening duration e.g. waves + river flow
- A method is presented for determining probable opening duration based on the geomorphic conditions

Abstract

The Victorian government spends tens of thousands of dollars artificially opening Intermittently Open/Closed Estuaries that become closed to the ocean. IOCE close when onshore deposition exceeds the capacity of the ebb-tidal prism to remove sediment from the entrance channel. Entrance closures often persist for months to years and can present a range of challenges for management. Despite providing short-term flooding relief, artificial openings do not always persist for more than a day or two as large quantities of sediment are redistributed landward. Infill of the channel commences after removal of the hydraulic head between the estuary and the ocean following lagoon drainage. The opening duration is then a function of the subsequent geomorphic conditions. This study uses historical records of entrance state and marine and fluvial conditions to develop a method of predicting the opening duration. Multiple linear regression models are developed from historic data and a Monte Carlo simulation is used to populate each model. The outcome is a predictive tool that managers can use to input both the current and forecast conditions to determine the probable entrance opening duration. This method will form a decision support tool for estuary entrance management that will have the capacity to learn and improve in future. It can also provide a complimentary module to work alongside existing entrance management strategies.

Keywords

Estuaries, Intermittently Open/Closed Estuaries, ICOLL, estuary management

Introduction

The majority of estuaries (93 %) on the open coast of Victoria, Australia, have entrances which periodically close to the ocean (McSweeney et al., 2017a). These estuaries are termed Intermittently Open/Closed Estuaries (IOCE). Entrance closure is a natural process occurring when the amount of sediment delivered onshore by waves exceeds the capacity of the ebb-tidal prism to remove it from the channel. Prolonged entrance closure can lead to flooding of low lying land/infrastructure and the deterioration of estuary water quality (Barton and Sherwood, 2004). To alleviate these issues, the entrances of many IOCE are artificially opened by excavator. Artificial entrance openings cost thousands of dollars at a time and often need to be implemented multiple times per year. A further issue is that IOCE entrances can sometimes close again within several days.

Recent research has improved our understanding of the processes that lead to IOCE opening and closure both internationally (Duong et al., 2016; Sadat-Noori et al., 2016; Slinger et al., 2016) and in Australia (Hinwood and McLean, 2015; McSweeney et al., 2017a; McSweeney et al., 2017b). Entrance closure occurs during low river flows when the ebb-tidal prism of the estuary is weak (Ranasinghe et al., 1999). This allows wave processes, specifically longshore drift or cross shore transport, to redistribute sediment within the channel without the counteracting outflow to remove it. The closure duration then becomes a function of the critical elevation of the beach berm vs the basin water level. If the basin water level surpasses the berm elevation, the estuary may incise a channel and open naturally to the ocean. In practice, many IOCE are artificially opened before this threshold is naturally surpassed. This is often due to the need to alleviate flooding of property surrounding the basin and to reconnect the estuary to the ocean to improve water quality. What remains crucial for estuary managers is being able to determine the likely opening duration under different geomorphic conditions. This will allow artificial openings to be undertaken at times when the estuary will not close again within hours-days.

The entrance opening duration refers to the time, in days, that the estuary remains open once an initial connection to the ocean has been established. The opening duration is a function of the offshore marine conditions (wave height, direction, period, and tidal range) in relation to the magnitude of river discharge and ebb-tidal currents (Ranasinghe and Pattaratchi, 2003). Following removal of the berm, the lagoon will drain to the ocean and progressively lower the water level of the basin to an elevation near mean sea level (Gordon, 1990). Once the lagoon has drained to a minimum depth, the hydraulic head between the lagoon and the ocean is removed. From this point onwards, the geomorphic conditions become the critical control on the rates of onshore vs offshore sediment transport and therefore the opening duration. To maintain an open entrance, energy is required from the catchment side (i.e. rainfall and river flow) to counteract marine depositional processes. It is often thought that an entrance channel with larger dimensions (i.e. depth, width, and cross-sectional area) will result in a longer opening duration however this was not the case as observed at entrance openings in Victoria (McSweeney et al., 2018). Being able to predict channel response to varying wave directions and heights relative to ebb-tidal energy is therefore a critical factor in predicting opening duration.

This study sets out to develop a method that can predict the probable entrance opening duration for multiple IOCE in Victoria by considering the historic influence of geomorphic conditions on the entrance condition. Entrance monitoring data now spans >10 years at some sites and presents a rare and valuable dataset to integrate into estuary entrance management. For each individual site, historic records of entrance opening duration and the corresponding geomorphic conditions are used to establish predictive models of opening duration. The physical processes at the time of opening, and those forecast in the subsequent days, can be input into each model to determine the change in predicted opening duration. A Monte Carlo simulation is used to populate each model and to produce monthly exceedance probabilities for each site. The proposed method can be used in conjunction with current entrance opening policy and existing decision support tools (e.g. Estuary Entrance Management Support System (EEMSS) - Arundel, 2006) to save estuary managers substantial amounts of money by reducing the need to reimplement openings.

Study area

The coast of Victoria, Australia

The open coast of Victoria, Australia, is microtidal with mixed semidiurnal tides. The spring tidal range varies between 0.80 - 1.60 m with higher tidal ranges occurring in embayments. Peak wave heights occur during May to September with wave height being largest from Cape Otway to the South Australian border (Figure 2a-d). Waves from the SW are dominant on the west coast (Nelson to Lorne) with a slight shift to the WSW from Lorne to Westernport, and then shifting again to the SW at Wilson’s Promontory (Figure 1a-d).

Victoria has a temperate climate with rainfall and river flow being both seasonally and interannually variable. Rivers draining coastal catchments experience peak flows during winter and spring, with low flows in summer (Puckridge et al., 1998). Rivers in the study region provide a negligible terrigenous sediment supply to the coast due to their small size (Davis Jr, 1989), the relatively low relief, and the potential for storage in the central basins of estuaries (Kench, 1999).

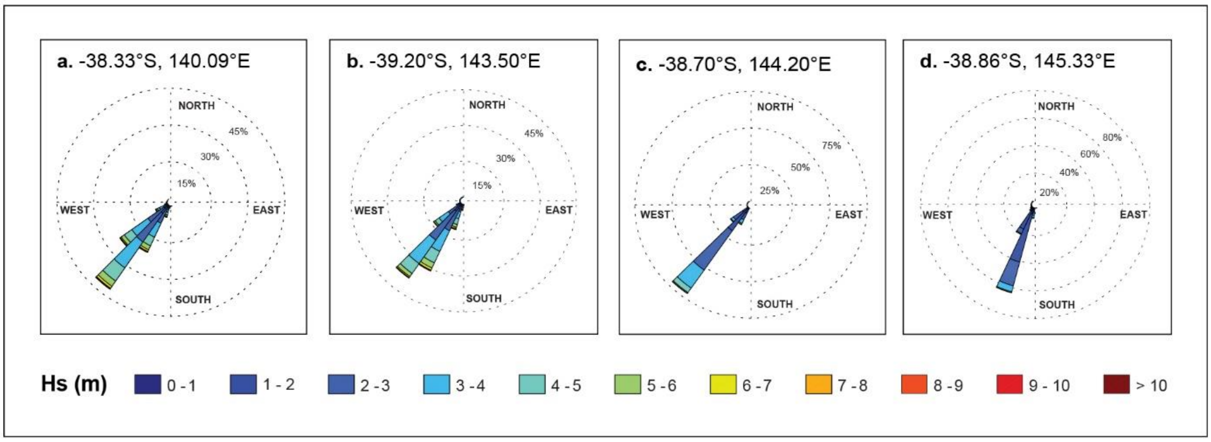

Figure 1a-d. Offshore directional wave roses showing deep-water mean daily significant wave height (Hs) (m) and wave direction (°N). Data hindcast from NOAA's WaveWatch III model (Aus 4m grid 1979 - 2009) (a) Far west - Portland; (b) Cape Otway; (c) Westernport; and (d) southern tip of Wilson's Promontory.

Study sites

This study specifically focuses on IOCE across the open coast of Victoria, Australia, that have detailed records of entrance condition (from Estuary Watch Victoria and EEMSS) (Figure 2; Table 1). Sites with available entrance condition data are concentrated west of Wilson's Promontory.

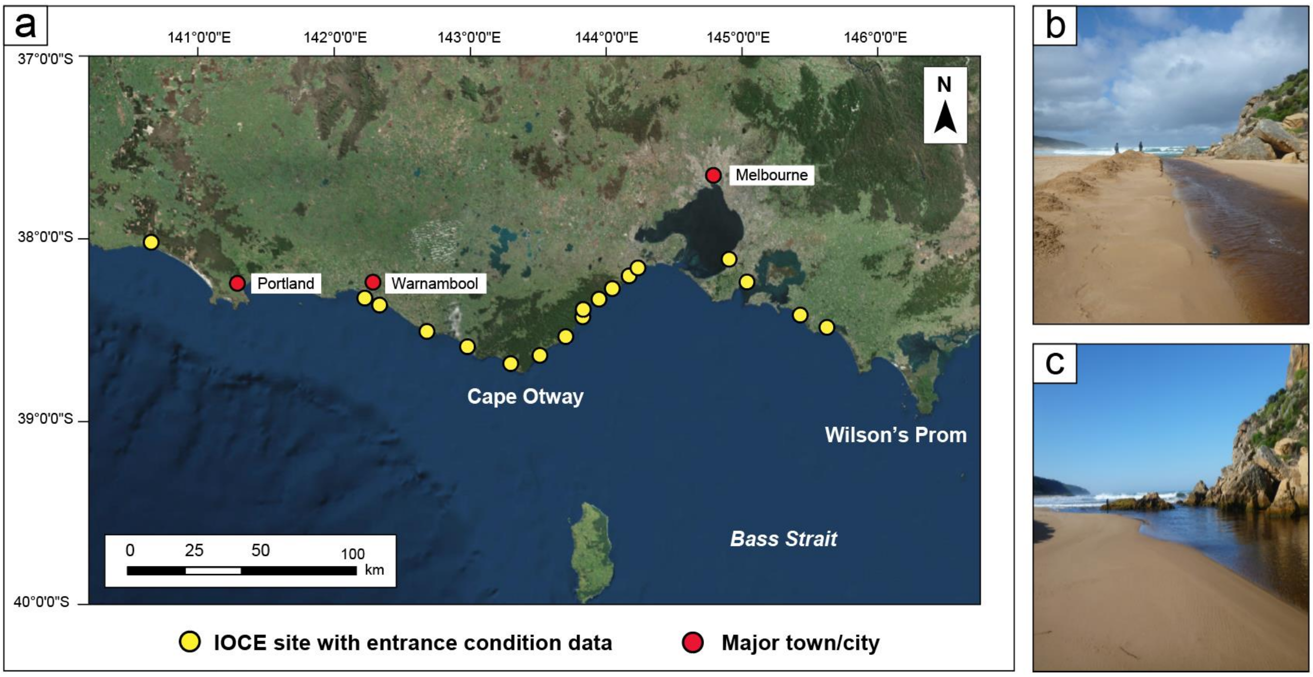

Figure 2a-c. a. Map of the coast of Victoria, Australia, showing IOCE with entrance conditions records (Estuary Watch Victoria + EEMSS) included in this study. Sites correspond with those in Table 1; b. Gellibrand River before artificial opening 2014; c. Gellibrand River six days after artificial opening 2014.

Table 1. Estuary sites included in this study which have entrance monitoring data. Sites correspond with those in Figure 1 (west to east). Some Estuary Watch sites which are inactive or have irregular (i.e. 1 year between observations) monitoring are excluded to maintain accuracy of MLR. Sites in bold will be targeted for field analysis in the future (e.g. surveying and monitoring of lagoon drainage rate following openings).

	Site (W - E)	Location	Duration of entrance monitoring	Artificial openings
1	**Glenelg River**	**-38.06, 140.99**	**2010 - 2018 (EEMSS; GHCMA)**	**Y**
2	Merri River	-38.40, 142.47	2013 - 2018 (Estuary Watch VIC)	Y
3	Hopkins River	-38.40, 142.51	2010 - 2018 (Estuary Watch VIC)	Y
4	**Curdies River**	**-38.61, 142.89**	**2014 - 2018 (Estuary Watch VIC)**	**Y**
5	**Gellibrand River**	**-38.71, 143.16**	**2007 - 2018 (Estuary Watch VIC)**	**Y**
6	Barham River	-38.76, 143.67	2008 - 2018 (Estuary Watch VIC)	Y
7	Wild Dog Creek	-38.74, 143.68	2008 - 2018 (Estuary Watch VIC)	N
8	Wye River	-38.63, 143.89	2007 - 2018 (Estuary Watch VIC)	N
9	St George River	-38.56, 143.98	2008 - 2018 (Estuary Watch VIC)	N
10	Erskine River	-38.53, 143.98	2008 - 2018 (Estuary Watch VIC)	?
11	Painkalac Creek	-38.47, 144.10	2007 - 2018 (Estuary Watch VIC)	Y
12	Anglesea River	-38.41, 144.19	2007 - 2018 (Estuary Watch VIC)	Y
13	Spring Creek	-38.34, 144.32	2007 - 2018 (Estuary Watch VIC)	Y
14	Thompson Creek	-38.30, 144.38	2009 - 2018 (Estuary Watch VIC)	Y
15	Balcombe Creek	-38.26, 145.02	2012 - 2018 (Estuary Watch VIC)	Y (by public)
16	Merricks Creek	-38.39, 145.15	2016 - 2018 (Estuary Watch VIC)	?
17	**Powlett River**	**-38.58, 145.51**	**2010 - 2018 (Estuary Watch VIC)**	**Y**
18	Wreck Creek	-38.65, 145. 70	2017 - 2018 (Estuary Watch VIC)	N

Methods

The methodology of the study, including inputs and outputs, is summarised in Figure 3.

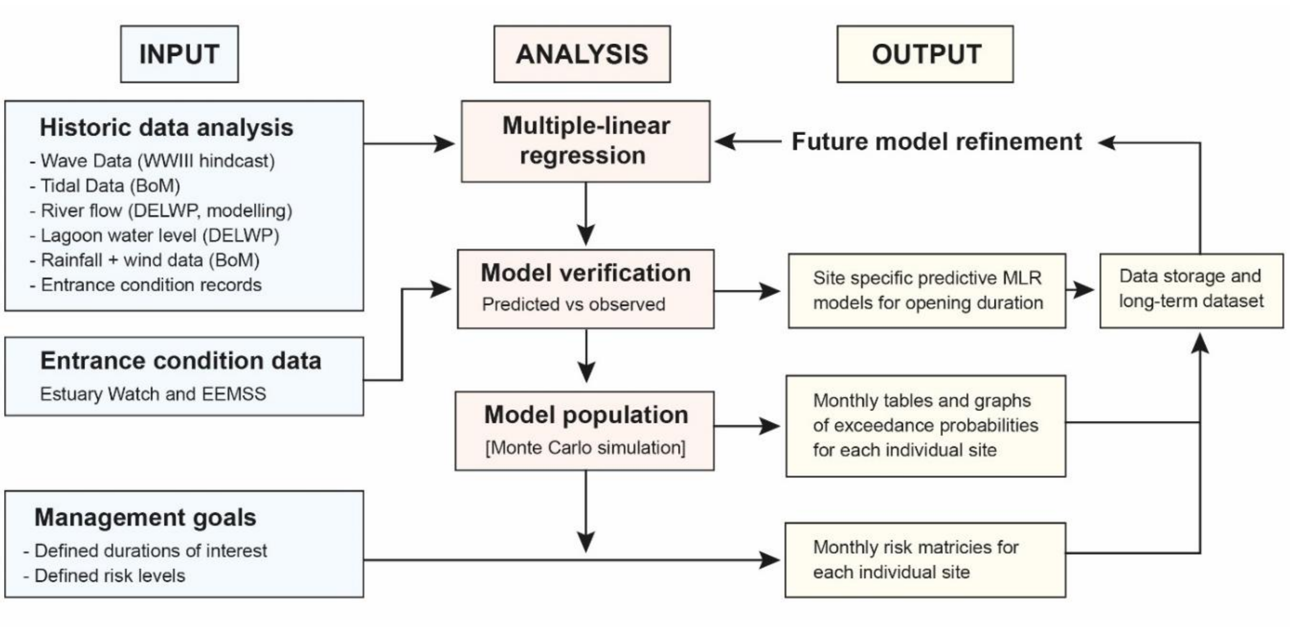

Figure 3. Study methodology and sequence of analysis including input data, analysis, and outputs.

Development of predictive models from historic data

The historic frequency and duration of IOCE entrance openings, along with the method of opening, were converted into a daily time series for each site using data from Estuary Watch Victoria and available EEMSS records. Mean daily geomorphic data were added to the time series and also obtained for the historical extent of data availability. Geomorphic data included: gauged daily river discharge and surface water elevation of the estuary basin (Department of Environment, Land, Water, and Planning (DELWP)), mean daily rainfall, wind speed and direction (Bureau of Meteorology (BoM)), and tidal elevation (BoM). All gauged data was taken from the closest available site to each estuary mouth. Wave direction (Dp), significant wave height (Hs), and period (Tp) were hindcast from NOAA's WAVEWATCH III model and extracted using MatLab R2015b software. Multiple linear regression (MLR) analysis was run in IMB SPSS Statistics v.24 to identify the influence of geomorphic predictor variables on the opening duration. All data were tested for autocorrelations and normality and MLR was run using a stepwise selection criteria. The MLR equations were then used to create predictive models to estimate the probable entrance opening duration at each site.

Monte Carlo analysis

The historic range of observed geomorphic conditions for each estuary were then partitioned into monthly data and fit with appropriate statistical distributions. For the data presented in this paper, the Weibull distribution was selected as it provided the best fit and is recognised as a good fit for many natural datasets which do not show a normal distribution (e.g. ocean wave heights (Battjes, 1972); wind (Sakar et al., 2011); rainfall (Wilks, 1989), wood in rivers (Stout, 2016); and flood risk (Apel et al., 2006)). Distributions were fit in MatLab R2015b using the maximum likelihood method. The predictive MLR model was then applied for each month to output a range of probable opening durations at each site. For each site, the predictive MLR model was then applied for each month using the historic range of data to output a range of probable opening durations. To populate the model, a Monte Carlo simulation was scripted in Python and run 1000 times for each month sampling at random from each monthly distribution. The Monte Carlo model was used to simulate the possible combination of geomorphic conditions that could occur in that month given the historic data. The output duration for each site were then quality checked against observed data from Estuary Watch and EEMSS.

Exceedance probabilities and risk matrices

The range of possible opening durations output from the Monte Carlo analysis were extracted and then presented as monthly exceedance probability (EEP) tables and graphs for each site. From the monthly exceedance probability data, risk matrices will be created for each site to show the probability that an opening will exceed a set duration (e.g. 1 week) if undertaken in each month of the year. Durations of interest will be informed by DELWP and coastal managers with a relevant risk level identified. The risk level refers to what specific exceedance probability could be considered to constitute a high, medium, or low risk.

Results

This paper will focus on three estuaries to exemplify the results and application of the research method: the Curdies, Gellibrand, and Powlett Rivers. From the available entrance condition data, Curdies Inlet has a mean opening duration of 96 days however this varies between 1 - 205 days (Table 2; Figure 4a). During 2016 - 2018, Curdies was artificially opened three times (Figure 4a). The two openings which persisted for a longer duration (>100 days) were undertaken prior to high fluvial discharge and rainfall in the catchment. The opening which did not last for <1 day was undertaken during periods of long Tp waves (17 sec) and low river flow (Figure 4a) - both being below the mean value for the site (Table 2). At the time of all closures, wave Dp was from the SW. The Powlett River has a mean opening duration of 267 days however more entrance monitoring data is being acquired to extend the data records for this site. Between 2016 - 2018, the Powlett River was artificially opened three times with the recorded openings lasting for >100 days duration. This was a function of opening prior to sustained high winter fluvial discharge and rainfall (Figure 4b). Closures occurred in accordance with a decrease in river discharge and a period of increased Hs and Tp, with a WSW Dp (Figure 4b). The Gellibrand shows the most variable range of opening durations (1 - 323 days) and the most frequent openings (both natural and artificial) annually (Figure 4c). Longer (>50 day) openings were associated with winter high river flows and increased rainfall. During winter, openings persisted irrespective of wave Hs, Tp, and Dp provided high river flow was maintained. All openings of a shorter duration (<1 month) were associated with a lack of sustained fluvial discharge and a SW wave Dp (Figure 3c). At all sites, opening at a higher water level did not result in a longer opening duration (Figure 4a-c).

Table 2. Geomorphic and entrance opening statistics: Curdies, Gellibrand, and Powlett Rivers. Data extent corresponds with entrance monitoring data: 2007 - 2018 for Curdies/Gellibrand, 2010 - 2018 for Powlett.

Site	Curdies River	Gellibrand River	Powlett River
Catchment area (km^2)	1245	1046	228
Mean daily rainfall (mm/day) (BoM)	2.40	2.91	2.69
Mean daily discharge (ML/day) (DELWP)	204.11	587.42	155.21
Mean daily Hs (m)	3.17	2.42	1.88
Mean daily Dp (°N)	226.67	211.96	238.76
Mean daily Tp (sec)	13.02	12.96	11.72
Mean opening duration (days)	96.17	82.60	267
Median opening duration (days)	90.01	28	269
Opening duration std. dev. (days)	70.26	101.02	190.7

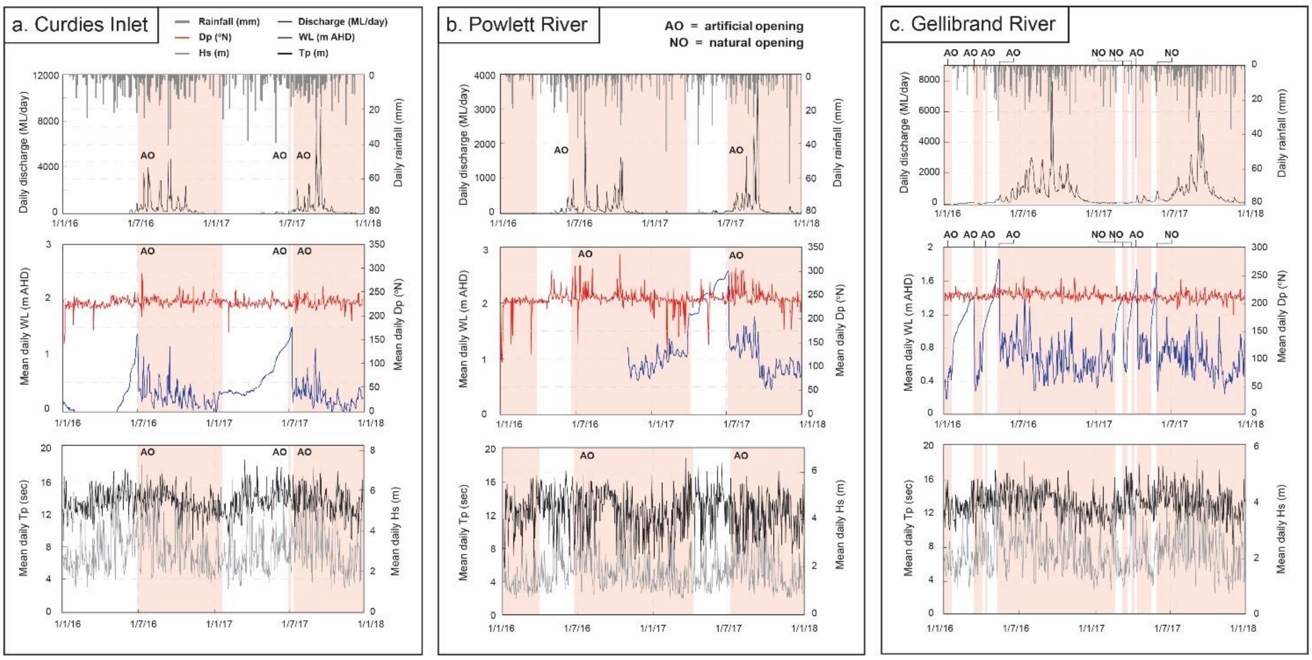

Figure 4. Marine and fluvial conditions and occurrence and timing of entrance openings 2016 - 2018. a. Curdies Inlet; b. Powlett River; c. Gellibrand River. WL is basin water level (m above AHD).

Equation 1 presents the predictive MLR model for the Gellibrand River estuary as this site had the highest frequency and duration of entrance monitoring. MLR had a R^2 value of 0.68 meaning it can account for 68 % of the variability in entrance opening duration by including these three parameters. MLR indicates that the wave Dp has a positive influence on increasing opening duration (i.e. more clockwise) and Hs and river discharge (Q) also have a positive influence. For each site, all models will be updated and refined as more data is available.

$$LogOpeningDuration = -7.882 + (0.034 * Dp) + (1.561 * LogHs) + (0.778 * LogQ) \quad \text{(Equation 1)}$$

EEP from Monte Carlo analysis are presented for the Gellibrand River (Figure 5a-d). The distribution of output opening durations for each month showed an infantile death rate decay curve indicating that despite the maximum output values being hundreds of days, most openings do not last >100 days (Figure 5a). When cross checking the Monte Carlo output durations with observed values (Estuary Watch data) there was R^2 of 0.69. The observed vs predicted values of the regression model show good accuracy for entrance openings lasting <200 days which represent the majority of openings in practice. Throughout the year, the lowest modelled opening durations occur during Dec - Apr with Feb having the lowest overall (mean = 27 days; median = 19 days) (Figure 5b). Figure 5c-d and Table 3 compare the output EEP for entrance opening exceeding 90 days, 1 month, 1 week, and 1 day for February and July respectively. There is a clear decrease in EEP for openings undertaken in Feb vs July which reflects the seasonal change in river flow.

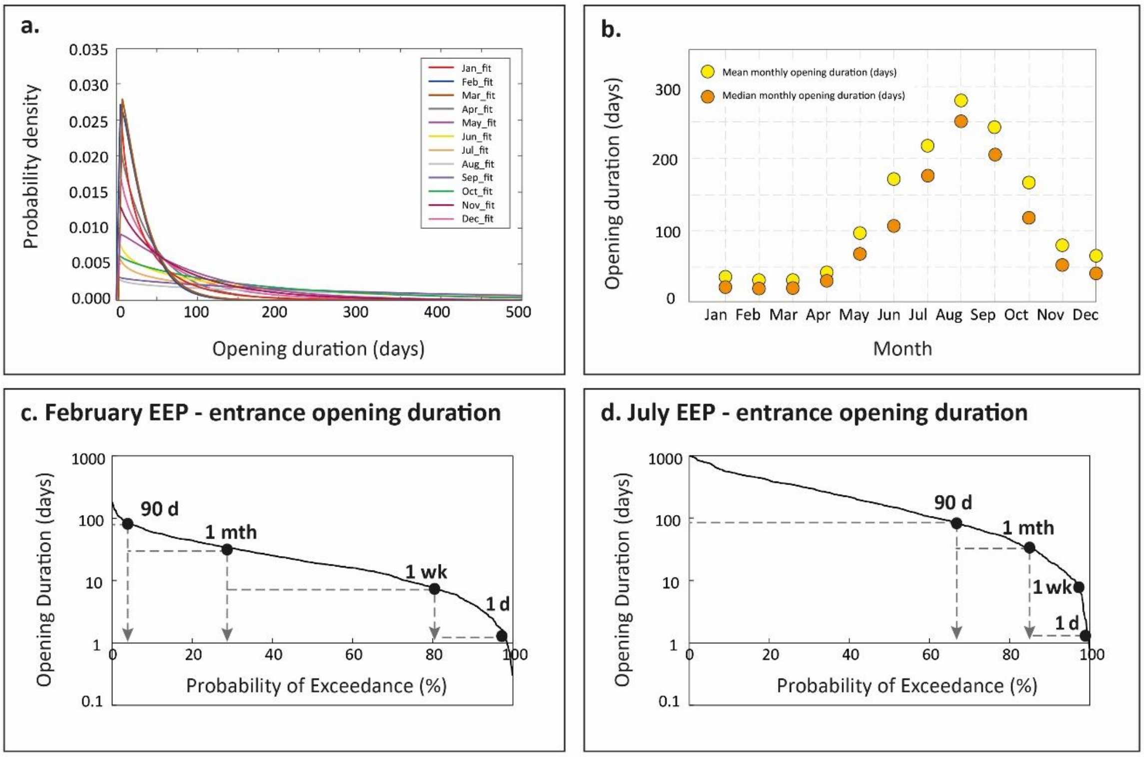

Figure 5. a. Distribution of modelled monthly opening durations for the Gellibrand River estuary; b. output mean and median monthly durations; c. February EEP (90 days, 1 month, 1 week, and 1 day); d. July EEP.

Discussion

The geomorphic conditions after lagoon drainage are most important in controlling the entrance opening duration. The basin water level (m AHD) at the time of entrance opening does not seem to influence the opening duration of IOCE and at each site to date, the basin water level was not a driving factor as indicated by MLR analysis of historic data. This is likely due to the loss of hydraulic head once the estuary has drained to the ocean. It is therefore important to be able to predict how different marine and fluvial conditions will control he rates of on vs offshore sediment transport once drainage has occurred. River discharge consistently shows a positive influence on entrance opening duration (e.g. Equation 1). This is due to the increased fluvial energy during high flows which is capable or adding to the ebb-tidal prism to transport sediment offshore from within the entrance channel (Cooper, 2001; Slinger et al., 2017). Undertaking artificial openings prior to periods of predicted sustained high river flow would be expected to increase the opening duration. Rainfall predictions could be used as proxies for this with catchment based flow modelling undertaken for each site.

The influence of wave Hs, Tp, and Dp is variable across individual sites as influenced by local swell conditions, nearshore bathymetry, winds, and rivermouth orientation (Barton and Sherwood, 2004). Higher waves tend to decrease the opening duration by moving more sediment onshore and into the entrance channel (Hoeksema et al., 2017; Ranasinghe et al., 1999). Under corresponding low river discharge, whereby the ebb-tidal prism is reduced, onshore sediment transport will result in a decrease in entrance cross-sectional area and an increase in berm elevation (McSweeney et al., 2018). At the Gellibrand River, wave height actually had a positive influence on entrance opening duration - although only during corresponding low flows. As swell height and fluvial discharge both increase during winter, winter high swells do not always close the entrance. During periods of winter high swells and river flow, storm surges were observed whereby the basin water level periodically increased upstream of the mouth for several days (e.g. Figure 4c ~15/07/16). At sites with smaller catchments, winter high swells may not have the same effect as there is still higher energy from the marine side with potential to overwash sediment into the basin and reestablish the berm. This illustrates the variable nature of estuary response to changing conditions and the importance of a site-by-site analysis.

A predictive tool, as presented in this study, is beneficial to estuary managers as it can help identify the probable opening duration given the current and forecast geomorphic conditions. The predictive MLR equations do not however account for change in conditions over time as they assume that the conditions remain constant. Substituting the parameter values within the model at any given time can predict the opening duration given the conditions for that day. For example, if discharge was to increase from the value on the first day of opening, an updated value should be input to account for this change. Forecast values can also be input to anticipate change over time and to assist with planning an implementation time for artificial openings. When combined with risk-based entrance management modules (e.g. EEMSS), there is potential to develop a world leading estuary entrance management system that also accounts for a range of geomorphic conditions.

Monte Carlo simulations have been used to populate each model and output a monthly range of possible opening durations (e.g. Figure 4a-d). An example of this data being converted to a risk matrix is presented in Table 3 for the Gellibrand River. This shows the change in opening duration EEP between February and July, being one of the two months with the most extreme change between mean opening duration. Risk classifications and durations of interest can be substituted by those selected by coastal managers.

Table 3. Example risk matrix for the Gellibrand River comparing the EEP for set opening durations between February and July. EEP correspond with the graphs in Figure 5c-d. All monthly data is also available.

Month	1 day	1 week	1 month	90 days
Feb	98.40	81.92	32.07	3.00
Jul	99.68	97.65	85.88	65.03

(red)	= Very high risk, <20 % chance entrance will remain open for set duration
(orange)	= High risk, 20 - 40 % chance entrance will remain open for set duration
(yellow)	= Moderate risk, 40 - 60 % chance entrance will remain open for set duration
(light green)	= Low risk, 60 - 80 % chance entrance will remain open for set duration
(dark green)	= Minimal risk, >80 % chance entrance will remain open for set duration

Conclusions

Estuary entrance dynamics are complex and dynamic due to the many interacting geomorphic processes at play and the variability in processes between individual sites. This study presents a method to predict estuary entrance opening duration given the current and forecast geomorphic conditions. MLR models and EEP can be used together to predict opening duration both spatially and temporally. A further step is to consider whether the rate of basin drainage can be predicted in a similar way to reduce the adverse environmental effects of rapid drainage (i.e. loss of DO and fish kills), or if this is a set function of channel expansion and knickpoint migration rates relative to outflow velocity (i.e. Parkinson and Stretch, 2007).

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